Pricing Credit Derivatives[1]

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Skora (1998) looked at how to price a credit default swap by explicitly constructing a hedge to the swap. Here we look at a variant of the credit default swap where the hedge for the swap structure is not obvious. In this case the price is obtained by first calculating implied default rates from bond prices and then by using these default rates in a lattice to obtain the price.

In Skora (1998) the market-maker agreed to make regular fixed payments, with the same frequency as the reference bond, to the investor for the duration of the swap. This was convenient for pricing the swap because, like the coupons of the reference bond, the regular fixed payments would cease upon default.

Here the market-maker agrees to make one fixed payment to the investor at the beginning of the swap. The exact time of default is important in order to properly present-value the future cashflows.

For simplicity, we will assume that the swap has a tenor of two years and that the coupon payments are annual. Also, we will assume that default can occur only at discrete times at the end of one year or of two years. In order to calculate the implied default rates we will need to calculate the full two-year term structure of both the riskless and credit risky rates.

The market data is as follows. The riskless term structure is flat at 5.00% for each of one and two years. The credit risky issuer of the reference bond has a term structure at one and two years at 8.00% and 8.25%, respectively. In particular, the two-year credit spread is 3.25%.

Assume a loss rate given default of 50% , so the recovery rate given default is 50%. Then the one- and two-year forward default rates are 5.56% and 6.50%, respectively. The first default rate is easily calculated using the formula given in the main text in the section on Default Models. The second default rate depends on the first default rate and so must be solved using a slightly more complicated formula. In any case, one can check that these default rates and loss rates price the one and two years bonds of 8.00% and 8.25%, respectively at par (Figure A).

[1] Submitted to *Credit Derivatives*, RISK Publications, 1998.

Also, for simplicity, let us assume that default is independent of the general level of interest rates. So future scenarios are simple to analyse: either the reference asset does not default, or it does default and it defaults at some time before the maturity of the trade.

There are several ways of implementing credit models – eg analytic formulas, Monte Carlo simulations, and lattices. The method selected depends on the credit model and the product that is being priced. For this example a collapsed lattice is sufficient. Since default is independent of the general level of riskless or risky interest rates, one may assume constant interest rates.

By the results in Skora (1998) the market-maker would expect the fair price for the credit default swap to be 3.25% per annum. We need to calculate the equivalent price when the market-maker pays it in full to the investor at the beginning of the transaction (Figure B).

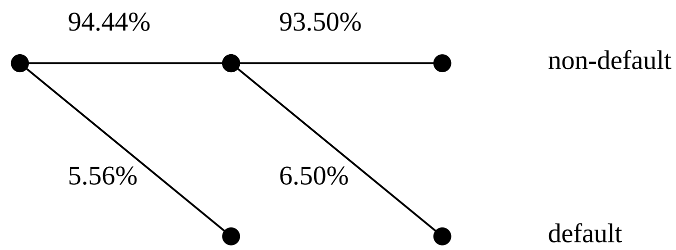
The calculation of the price is illustrated in Figure C. Recall that the price is the expected value of the present value of future cashflows. The expectation is taken with respect to the risk-neutral default probabilities, which were calculated above. The present value is calculated with respect to the riskless rate of 5.00%, which corresponds to a one-period discount factor of 0.9524. The simple arithmetic gives a price of 5.883%. This means the market-maker makes an upfront payment to the investor at the beginning of the swap which is 5.88% of the par value of the reference asset.

Notice that in the above example the lattice is calibrated to the term structure of bond yields. This means that it correctly prices a long bond position. (The product is also naturally long the credit risk.) If one were to hedge the product, one would have to *short* the bond – and shorting is more expensive. Thus if one actually intended to hedge the position by shorting bonds, then the pricing model should be calibrated to the short bond prices and this would naturally give a higher price for the product.

Finally, the loss rate assumption influences the price. A different loss assumption would change the default rates and, therefore, change the non-default rates. This in turn would change the present value of the two coupons of 3.25%.

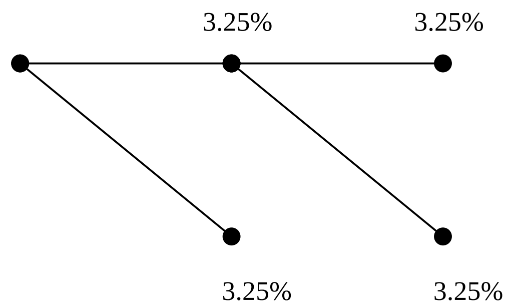
Bibliography

1. Richard K. Skora, 1998, "The Credit Default Swap," working paper.



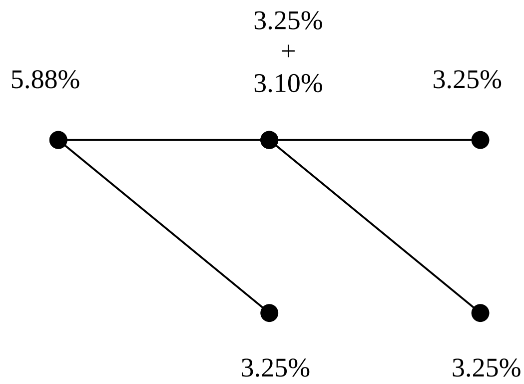
Non-Default and Default Probabilities

Figure X



Cash Flows

Figure Y



Price

Figure Z

Skora & Company Inc. is a new credit risk management advisory firm which offers numerous products and services to successfully manage, trade, sell, model and structure credit risk. It has the expertise and experience to support its clients at every stage of their business development.

Skora & Company has already helped financial and non-financial institutions set up profitable credit derivatives trading desks, build cutting edge portfolio credit risk management systems, and design efficient credit risk/return performance analytics.

Richard K. Skora is the founder of Skora & Company. He worked in the credit risk management since 1992. He also traded various exotic credit derivatives including default swaps, default options, and basket swaps.

Mr. Skora received a B.S. in mathematics from The University of Illinois in Champaign-Urbana and a Ph.D. in mathematics from The University of Texas in Austin. He held academic positions at The Institute for Advanced Study in Princeton and Columbia University in New York.

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